NAG Toolbox for MATLAB

f07hf

1 Purpose

f07hf computes a diagonal scaling matrix S intended to equilibrate a real n by n symmetric positive-definite band matrix A, with bandwidth $(2k_d + 1)$, and reduce its condition number.

2 Syntax

```
[s, scond, amax, info] = f07hf(uplo, kd, ab, 'n', n)
```

3 Description

f07hf computes a diagonal scaling matrix S chosen so that

$$s_j = 1/\sqrt{a_{jj}}.$$

This means that the matrix B given by

$$B = SAS,$$

has diagonal elements equal to unity. This in turn means that the condition number of B, $\kappa_2(B)$, is within a factor n of the matrix of smallest possible condition number over all possible choices of diagonal scalings (see Corollary 7.6 of Higham 2002).

4 References

Higham N J 2002 *Accuracy and Stability of Numerical Algorithms* (2nd Edition) SIAM, Philadelphia

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo – string**

Indicates whether the upper or lower triangular part of A is stored in the array **ab**, as follows:

uplo = 'U'

The upper triangle of A is stored.

uplo = 'L'

The lower triangle of A is stored.

Constraint: **uplo** = 'U' or 'L'.

2: **kd – int32 scalar**

k_d, the number of superdiagonals of the matrix A if **uplo** = 'U', or the number of subdiagonals if **uplo** = 'L'.

Constraint: **kd** ≥ 0.

3: **ab**(**ldab**,$*$) **– double array**

The first dimension of the array **ab** must be at least **kd** + 1

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The upper or lower triangle of the symmetric positive-definite band matrix A whose scaling factors are to be computed.

The matrix is stored in rows 1 to $k_d + 1$, more precisely,

if **uplo** = 'U', the elements of the upper triangle of A within the band must be stored with element A_{ij} in $\mathbf{ab}(k_d + 1 + i - j, j)$ for $\max(1, j - k_d) \leq i \leq j$;
if **uplo** = 'L', the elements of the lower triangle of A within the band must be stored with element A_{ij} in $\mathbf{ab}(1 + i - j, j)$ for $j \leq i \leq \min(n, j + k_d)$.

Only the elements of the array **ab** corresponding to the diagonal elements of A are referenced. (Row $(k_d + 1)$ of **ab** when **uplo** = 'U', row 1 of **ab** when **uplo** = 'L'.)

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The second dimension of the array **ab**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

ldab

5.4 Output Parameters

1: **s**$(*)$ **– double array**

Note: the dimension of the array **s** must be at least $\max(1, \mathbf{n})$.

If **info** = 0 on exit, **s** contains the diagonal elements of the scaling matrix S.

2: **scond – double scalar**

If **info** = 0 on exit, **scond** contains the ratio of the smallest value of $\mathbf{s}(i)$ to the largest value of $\mathbf{s}(i)$. If **scond** ≥ 0.1 and **amax** is neither too large nor too small, it is not worth scaling by S.

3: **amax – double scalar**

$\max |a_{ij}|$. If **amax** is very close to overflow or underflow, the matrix A should be scaled.

4: **info – int32 scalar**

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **kd**, 4: **ab**, 5: **ldab**, 6: **s**, 7: **scond**, 8: **amax**, 9: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** $= i$, the ith diagonal element of A is not positive (and hence A cannot be positive-definite).

7 Accuracy

The computed scale factors will be close to the exact scale factors.

8 Further Comments

The complex analogue of this function is f07ht.

9 Example

```
uplo = 'U';
kd = int32(1);
ab = [0, 26800000000, -23900000000, -2.22;
     5.49, 5.63e+20, 2.6, 5.17];
[s, scond, amax, info] = f07hf(uplo, kd, ab)
```

```
s =
    0.4268
    0.0000
    0.6202
    0.4398
scond =
   6.7957e-11
amax =
   5.6300e+20
info =
           0
```